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Research on Quality and Safety Strategies of Cosmetics under Platform Supervision Mechanism

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ABSTRACT

Since the reform and opening up, China's sustained economic development, coupled with the rise of new consumption patterns such as e-commerce shopping, live-streaming marketing and overseas purchasing agents, has fueled the continuous prosperity of the cosmetics industry. As daily consumer goods, cosmetics quality is directly related to people's life and health. Incidents involving substandard cosmetics not only damage corporate reputation and interests but also endanger consumers' health and lives, further triggering public distrust and questioning of the entire cosmetics industry and regulatory authorities. While the booming cosmetics industry provides people with numerous benefits and conveniences, it also poses many new challenges to regulators. From the perspective of cosmetics quality and safety supervision, and considering information asymmetry and bounded rationality, this paper constructs a tripartite evolutionary game model among cosmetics manufacturers, beauty reviewers, and platform regulators, and analyzes the mutual influence mechanism of strategy choices among different stakeholders and the evolutionary trend of each participant's strategic decisions.

KEYWORDS

Quality Supervision; Evolutionary Game; Strategy Selection.

1. INTRODUCTION

Since the reform and opening up, China's economy has maintained steady growth, people's living standards have improved markedly, and purchasing power has continued to rise. As the world's most populous country, China has seen a growing demand for cosmetics. In the era of the "lookism economy", people, especially female beauty reviewers, pay increasing attention to personal appearance and dressing. Cosmetics have become daily necessities, and consumers have put forward higher requirements for cosmetic products.

The cosmetics industry is generally characterized by high R&D costs, long test cycles and expensive product testing. It is difficult and time-consuming to produce high-quality cosmetics, while counterfeit and inferior cosmetics are easy to make yet yield huge profits. Amid fierce market competition, some enterprises resort to practices such as passing off shoddy goods as qualified ones to boost product sales and seek quick gains. Such behaviors seriously damage consumers' interests and health and disrupt market order. Rent-seeking activities also allow inferior cosmetics to enter the market, threatening people's personal safety and property.

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Meanwhile, marketing methods in the cosmetics industry are constantly evolving, and these emerging approaches have set higher standards for supervision. Regulators must keep pace with market changes and strengthen supervision to guarantee the quality and safety of cosmetics and protect consumers' rights and interests. Relevant work includes reviewing cosmetic ingredients, supervising the authenticity of marketing promotions, and promptly handling and preventing incidents involving substandard cosmetics. These measures can enhance consumers' trust in the cosmetics industry and promote its sound development.

2. MODEL CONSTRUCTION AND SOLUTION

Model Assumptions

Assumption 1:The cosmetic manufacturer is defined as Player 1, the beauty reviewer as Player 2, and the platform regulatory authority as Player 3. All three participants are bounded rational subjects.The strategy spaces of the participants are set as follows:The strategy space of the cosmetic manufacturer: S_1= (Produce high-quality products, Produce inferior products);The strategy space of the beauty reviewer: S_2= (Provide truthful feedback, Provide false feedback);The strategy space of the platform regulatory authority: S_3= (Implement strict supervision, Implement loose supervision).

Assumption 2:Let $x(0 \le x \le 1)$ be the probability that the cosmetic manufacturer chooses to produce high-quality products, then the probability of producing inferior products is $1-x$.Let $y(0 \le y \le 1)$ be the probability that the beauty reviewer chooses to provide truthful feedback, then the probability of providing false feedback is$1-y$.Let $z(0 \le z \le 1)$ be the probability that the platform regulatory authority chooses to implement strict supervision, then the probability of implementing loose supervision is $1-z$.

Assumption 3:The production cost for high-quality cosmetics is C_h, and the production cost for inferior cosmetics is C_l, where $C_h > C_l > 0$. The sales revenue of the cosmetic manufacturer is R.The manufacturer pays rent-seeking cost Cr to bribe beauty reviewers, including cash rebates, coupons and other forms, and $C_r \le C_h - C_l$. If the manufacturer produces inferior products, it shall pay liquidated damages F_w. Among the compensation, the proportion α goes to the beauty reviewer, and the remaining proportion $(1-\alpha)$ is paid to the platform regulatory authority.

Assumption 4:The utility obtained by the beauty reviewer from using high-quality cosmetics is U_h, and the utility from using inferior cosmetics is $-U_l$.The cost for the beauty reviewer to deliver truthful feedback based on objective and rational criteria is C_{t1}, and the cost for delivering false feedback is C_{t2}. Since false feedback requires more financial, material and time resources, we have $C_{t2} > C_{t1} > 0$.

Assumption 5:High-quality cosmetics produced by manufacturers contribute to social stability and economic development, bringing social benefit g to the platform.When the platform authority implements strict supervision, it will win public recognition, with the corresponding supervision cost being Cg. When loose supervision is adopted, inferior products will enter the market and harm public health. To maintain social stability and economic order, the platform needs to bear governance cost Dg.Dereliction of supervision will lead to accountability from higher authorities and administrative penalty T_g, where $T_g > C_g$.

Table 1. Mixed-strategy Game Payoff Matrix of Cosmetic Manufacturers, Beauty Reviewers and Platform Regulatory Authorities

Strategy Selection		beauty reviewers	platform regulatory authorities	
			stringent supervision	loose supervision
Cosmetic Manufacturing Enterprises	produce high-quality products	authentic feedback	$R - C_h, U_h - C_{t1}, -C_g + g$	$R - C_h, U_h - C_{t1}, g$
		false feedback	$R - C_h, U_h - C_{t2}, -C_g + g$	$R - C_h, U_h - C_{t2}, g$
	produce inferior products	authentic feedback	$R - C_l - C_r - F_w,$ $-U_l - C_{t1} + \alpha F_w,$ $-C_g + (1-\alpha)F_w + C_r$	$R - C_l - C_r - F_w,$ $-U_l - C_{t1} + F_w,$ $-T_g + C_r$
		false feedback	$R - C_l - C_r - F_w,$ $-U_l - C_{t2} + C_r,$ $-C_g + F_w - D_g$	$R - C_l - C_r,$ $-U_l - C_{t2} + C_r,$ $-T_g - D_g$

3. MODEL ANALYSIS

3.1 Stability Analysis of Strategies for Cosmetic Manufacturers

The expected payoff E11 when cosmetic manufacturers choose the strategy of "producing high-quality products" is expressed as follows:

$$E_{11} = yz(R - C_h) + y(1-z)(R - C_h) + (1-y)z(R - C_h) + (1-y)(1-z)(R - C_h)$$

The expected payoff E12 of cosmetic manufacturers choosing the strategy of "producing inferior products" is expressed as follows:

$$E_{12} = yz(R - C_l - C_r - F_w) + y(1-z)(R - C_l - C_r - F_w) + (1-y)z(R - C_l - C_r - F_w) + (1-y)(1-z)(R - C_l - C_r)$$

The average expected payoff E¯1 of cosmetic manufacturers is expressed as follows:

$$\bar{E}_1 = xE_{11} + (1-x)E_{12}$$

The replicator dynamic equation for the strategy selection of cosmetic manufacturers is expressed as follows:

$$F(x) = \frac{dx}{dt} = x(E_{11} - \bar{E}_1) = x(1-x)[C_l + C_r - C_h + y(1-z)F_w + zF_w]$$

By calculating the partial derivative of the replicator dynamic equation F(x) for the probability that cosmetic manufacturers choose to produce high-quality cosmetics, we obtain:

$$\frac{d(F(x))}{dx} = (1-2x)[C_l + C_r - C_h + y(1-z)F_w + zF_w]$$
$$G(y,z) = C_l + C_r - C_h + y(1-z)F_w + zF_w$$

Corollary: When the probability of strict supervision adopted by the platform regulatory authority is relatively high, the probability that cosmetic manufacturers choose to produce high-quality cosmetics will stabilize at 1. Conversely, if the platform authority implements strict supervision with a low probability, the strategy of producing high-quality cosmetics will stabilize at 0 for cosmetic enterprises.

Proof: Let $G(y,z) = 0$ then $z = \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$. Since $\frac{\partial G(y,z)}{\partial z} > 0$, $G(y,z)$ is an increasing function with respect to z.If $z > \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$, we have $\frac{\partial F(x)}{\partial x}|_{x=0} > 0$ and $\frac{\partial F(x)}{\partial x}|_{x=1} < 0$.

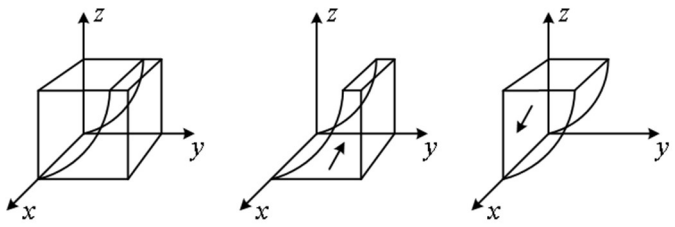
On the contrary, if $z < \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$, we have $\frac{\partial F(x)}{\partial x}|_{x=0} < 0$ and $\frac{\partial F(x)}{\partial x}|_{x=1} > 0$.

The reaction function for the probability x that cosmetic manufacturers choose to produce high-quality products is as follows:

$$x = \begin{cases} 0 & if\ z < \dfrac{yF_w + C_l + C_r - C_h}{F_w(y-1)} \\ [0,1] & if\ z = \dfrac{yF_w + C_l + C_r - C_h}{F_w(y-1)} \\ 1 & if\ z > \dfrac{yF_w + C_l + C_r - C_h}{F_w(y-1)} \end{cases}$$

It can be concluded that if $z = \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$, all states are in a neutral equilibrium. If $z > \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$, then $x^* = 1$ is the evolutionary stable strategy. If $z < \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$, then $x^* = 0$ is the evolutionary stable strategy.When $z > \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)}$ and $x = 1$ serves as the evolutionary stable strategy, it indicates that when the platform regulatory authority adopts strict supervision, cosmetic manufacturers will attach importance to product quality and safety to pass inspections, and accordingly raise the probability of producing high-quality cosmetics. On the contrary, under loose supervision, manufacturers tend to take opportunistic actions and cut production costs, thus opting to produce inferior cosmetics.

The diagram of the evolutionary stable strategy for cosmetic manufacturers choosing to produce high-quality products is presented below.



$x = x^*$ $x < x^*$ $x > x^*$

Fig. 1 Evolutionary Stable Strategy Diagram for Cosmetic Producers

The volume V_{I_m} of Region I_m represents the probability that cosmetic manufacturers choose to produce high-quality cosmetics, and the volume V_{II_m} of Region II_m denotes the probability of the same strategy.

$$V_{I_m} = \iint \frac{yF_w + C_l + C_r - C_h}{F_w(y-1)} dydz$$

$$V_{II_m} = 1 - V_{I_m}$$

Corollary: The probability that cosmetic manufacturers produce high-quality products is positively correlated with C_r, F_w and C_l, and negatively correlated with C_h.

Proof: Calculate the partial derivatives of V_{II_m} with respect to C_r, F_w, C_l and C_h, respectively. The results show that $\frac{\partial V_{II_m}}{\partial C_r} > 0, \frac{\partial V_{II_m}}{\partial F_w} > 0, \frac{\partial V_{II_m}}{\partial C_l} > 0$ and $\frac{\partial V_{II_m}}{\partial C_h} < 0$. This indicates that the increase in the penalty F_w for producing inferior products and the rent-seeking cost C_r will drive enterprises to produce high-quality products to cut down total expenditures. Nevertheless, when the production cost *Ch* of high-quality products rises to a certain level, some enterprises, tempted by short-term profits, will neglect brand building and market reputation, and thereby reduce the likelihood of manufacturing high-quality cosmetics.

Conclusion: To incentivize enterprises to produce high-quality products, regulators should strengthen supervision and penalties, strictly enforce product quality control, promote communication and cooperation to curb corporate rent-seeking, and fundamentally block illegal rent-seeking gains of beauty reviewers. Meanwhile, competent authorities may offer preferential policies for manufacturers of high-quality products, such as raw material discounts and subsidies for factory and equipment rents, so as to reduce production costs and guide enterprises to make rational strategic choices.

3.2 Stability Analysis of Beauty Reviewers' Strategies

The expected payoff E_{21} when beauty reviewers adopt the strategy of providing truthful feedback is expressed as follows:

$$E_{21} = xz(U_h - C_{t1}) + x(1-z)(U_h - C_{t1}) + (1-x)z(-U_l - C_{t1} + \alpha F_w) + (1-x)(1-z)(-U_l - C_{t1} + F_w)$$

The expected payoff E_{22} when beauty reviewers adopt the strategy of providing false feedback is expressed as follows:

$$E_{22} = xz(U_h - C_{t2}) + x(1-z)(U_h - C_{t2}) + (1-x)z(-U_l - C_{t2} + C_r) + (1-x)(1-z)(-U_l - C_{t2} + C_r)$$

The average expected payoff $\bar{E}_2$ of beauty reviewers is given below:

$$\bar{E}_2 = yE_{21} + (1-y)E_{22}$$

The replicator dynamic equation for the strategy selection of beauty reviewers is as follows:

$$F(y) = \frac{dy}{dt} = y(E_{21} - \bar{E}_2) = y(1-y)\{C_{t2} - C_{t1} - C_r + xC_r + [1 + (\alpha - 1)z](1-x)F_w\}$$

Compute the first-order derivative of F(y) with respect to y:

$$\frac{d(F(y))}{dy} = (1-2y)\{C_{t2} - C_{t1} - C_r + xC_r + [1 + (\alpha - 1)z](1-x)F_w\}$$

Let $H(x,z,\alpha) = C_{t2} - C_{t1} - C_r + xC_r + [1 + (\alpha - 1)z](1-x)F_w$ Setting $H(x,z,\alpha) = 0$, we can derive the solution:

When $x = \frac{[1+(\alpha-1)z]F_w - C_r - C_{t1} + C_{t2}}{[1+(\alpha-1)z]F_w - C_r}$,we have $\frac{d(F(y))}{dy} \equiv 0$. For cases where x does not equal the above value, further discussion on α is required.

Corollary: When the compensation paid to beauty reviewers is relatively high, if cosmetic manufacturers have a low probability of producing high-quality cosmetics, the probability that beauty reviewers choose to provide truthful feedback will stabilize at 1. On the contrary, if manufacturers are highly likely to produce high-quality cosmetics, the probability of beauty reviewers adopting truthful feedback will stabilize at 0.

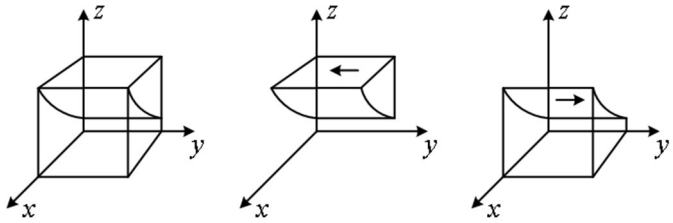
Proof: Let $\frac{\partial(H(x,z,\alpha))}{\partial x} = 0$, and we obtain $\alpha = \left(\frac{C_r}{F_w} - 1\right)\frac{1}{z} + 1$.Since $\frac{\partial^2(H(x,z,\alpha))}{\partial x \partial \alpha} < 0$, it follows that if $\alpha < \left(\frac{C_r}{F_w} - 1\right)\frac{1}{z} + 1$,then $\frac{\partial(H(x,z,\alpha))}{\partial x} > 0$, meaning $H(x,z,\alpha)$ is an increasing function of x.

When $x > \frac{[1+(\alpha-1)z]F_w - C_r - C_{t1} + C_{t2}}{[1+(\alpha-1)z]F_w - C_r}$ and $H(x,z,\alpha) > 0$,so $\frac{\partial F(y)}{\partial y}|_{y=0} > 0$, $\frac{\partial F(y)}{\partial y}|_{y=1} < 0$.Conversely, when $x < \frac{[1+(\alpha-1)z]F_w - C_r - C_{t1} + C_{t2}}{[1+(\alpha-1)z]F_w - C_r}$ and H(x,z,α)<0, $so \frac{\partial F(y)}{\partial y}|_{y=0} < 0$, $\frac{\partial F(y)}{\partial y}|_{y=1} > 0$.In summary, when $\alpha > \left(\frac{C_r}{F_w} - 1\right)\frac{1}{z} + 1$,the reaction function for the probability y that beauty reviewers choose truthful feedback (an externally increasing function) is defined as:

$$y=\begin{cases}0 & if x<\dfrac{[1+(\alpha-1)z]F_w-C_r-C_{t1}+C_{t2}}{[1+(\alpha-1)z]F_w-C_r}\\ [0,1] & if x=\dfrac{[1+(\alpha-1)z]F_w-C_r-C_{t1}+C_{t2}}{[1+(\alpha-1)z]F_w-C_r}\\ 1 & if x>\dfrac{[1+(\alpha-1)z]F_w-C_r-C_{t1}+C_{t2}}{[1+(\alpha-1)z]F_w-C_r}\end{cases}$$

It can be concluded that if $x=\frac{[1+(\alpha-1)z]F_w-C_r-C_{t1}+C_{t2}}{[1+(\alpha-1)z]F_w-C_r}$, all states remain in neutral equilibrium. If x is greater than the above value, y=1 is the evolutionary stable strategy. If x is less than the above value, $y=0$ is the evolutionary stable strategy.When x exceeds the threshold and $y=1$ becomes the evolutionary stable strategy, it indicates that a high proportion of high-quality products brings good experience, pleasure and satisfaction to beauty reviewers, who accordingly deliver truthful feedback based on product quality. In contrast, when manufacturers are unlikely to produce high-quality products, user experience declines dramatically. To reduce complaints, manufacturers will intensify rent-seeking behaviors such as issuing coupons and offering discounts, which raises the likelihood that beauty reviewers accept bribes and post false feedback.

The diagram of the evolutionary stable strategy for beauty reviewers choosing to provide truthful feedback is shown below.



$y=y^*$ $y<y^*$ $y>y^*$

Fig. 2 Evolutionary Stable Strategy Diagram for Review Influencers

The volume V_{I_c} of Region I_c represents the probability that beauty reviewers choose to provide truthful feedback, and the volume V_{II_c} of Region II_c represents the probability of providing false feedback.

$$V_{I_c}=\iint\frac{[1+(\alpha-1)z]F_w-C_r-C_{t1}+C_{t2}}{[1+(\alpha-1)z]F_w-C_r}dydz$$
$$=\frac{(-C_{t1}+C_{t2})ln(\alpha F_w-C_r)+(C_{t1}-C_{t2})ln(-C_r+F_w)+F_w(\alpha-1)}{F_w(\alpha-1)}$$
$$V_{II_c}=1-V_{II_c}$$

Corollary: The probability that beauty reviewers provide truthful feedback to cosmetic manufacturers is positively correlated with F_w (the compensation paid by manufacturers for producing inferior products after being reported) and C_{t2} (the cost of false feedback), and negatively correlated with C_r (manufacturers' rent-seeking cost) and C_{t1} (the cost of truthful feedback).

Proof: We calculate the partial derivatives of VIc with respect to F_w,C_{t2},C_r and C_{t1} respectively, and obtain:$\frac{\partial V_{I_c}}{\partial F_w} > 0$, $\frac{\partial V_{I_c}}{\partial C_{t2}} > 0$, $\frac{\partial V_{I_c}}{\partial C_r} < 0$,$\frac{\partial V_{I_c}}{\partial C_{t1}} < 0$An increase in the penalty for inferior products motivates beauty reviewers to provide truthful feedback so as to avoid losses. Nevertheless, if the rent-seeking benefits offered by manufacturers reach a certain level, beauty reviewers will likely take risks to obtain illegal gains.

Conclusion: The strategic choices of beauty reviewers are closely linked to those of the other two participants. To encourage truthful product reviews and build a fair and stable market environment, platform regulators should strengthen supervision and raise penalties. Meanwhile, it is necessary to guide beauty reviewers to establish rational consumption concepts, optimize the allocation of market resources and improve economic efficiency.Regulators can also implement reward and punishment mechanisms: penalize beauty reviewers who accept corporate rent-seeking and reward those who fulfill their obligations honestly. These measures can improve the utilization efficiency of social resources and promote high-quality economic development.

3.3 Stability Analysis of Platform Regulatory Authorities' Strategies

The expected payoff E_{31} when the platform regulatory authority adopts the strict supervision strategy is expressed as follows:

$$E_{31} = xy(-C_g + g) + x(1-y)(-C_g + g) + (1-x)y(-C_g + (1-\alpha)F_w + C_r) + (1-x)(1-y)(-C_g + F_w - D_g)$$

The expected payoff E_{32} when the platform regulatory authority adopts the loose supervision strategy is expressed as follows:

$$E_{32} = xy(g) + x(1-y)(g) + (1-x)y(-T_g + C_r) + (1-x)(1-y)(-T_g - D_g)$$

The average expected payoff $\bar{E}_3$ of the platform regulatory authority is given below:

$$\bar{E}_3 = zE_{31} + (1-z)E_{32}$$

The replicator dynamic equation for the strategy selection of the platform regulatory authority is as follows:

$$F(z) = \frac{dz}{dt} = z(E_{31} - \bar{E}_3) = z(1-z)\left[(\alpha y - 1)(x-1)F_w - xT_g - C_g + T_g\right]$$

Take the first derivative of F(z) with respect to z:

$$\frac{d(F(z))}{dz} = (1-2z)\left[(\alpha y - 1)(x-1)F_w - xT_g - C_g + T_g\right]$$

Let $K(x,y,\alpha)=(\alpha y-1)(x-1)F_w-xT_g-C_g+T_g$,Set $K(x,y,\alpha)=0$, and we obtain the solution:

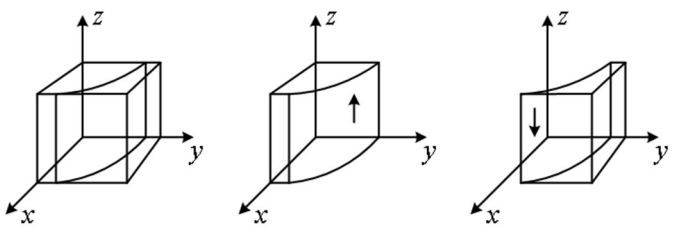
When $x=\frac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}$ and $\frac{d(F(z))}{dz}\equiv 0$.If x does not equal the above value, further discussion on α is required.

Corollary: When the compensation paid to beauty reviewers is relatively high, the probability that the platform regulatory authority chooses strict supervision will stabilize at 0 if cosmetic manufacturers have a low probability of producing high-quality cosmetics. On the contrary, if manufacturers are highly likely to produce high-quality products, the probability of strict supervision will stabilize at 1.

Proof: Let $\frac{\partial(K(x,y,\alpha))}{\partial x}=0$, then $\alpha=\frac{F_w+T_g}{yF_w}$. Since $\frac{\partial^2(K(x,y,\alpha))}{\partial x\partial\alpha}=yF_w>0$, it can be concluded that if $\alpha<\frac{F_w+T_g}{yF_w}$ and $\frac{d(K(x,y,\alpha))}{dx}<0$, which means $K(x,y,\alpha)$ is a decreasing function of x.When $x>\frac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}$ we get $K(x,y,\alpha)<0$, so $\frac{\partial F(z)}{\partial z}|_{z=0}<0$, $\frac{\partial F(z)}{\partial z}|_{z=1}>0$ Conversely, when $x<\frac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}$,we get $K(x,y,\alpha)>0$, so $\frac{\partial F(z)}{\partial z}|_{z=0}>0$, $\frac{\partial F(z)}{\partial z}|_{z=1}<0$ In summary, when $\alpha<\frac{F_w+T_g}{yF_w}$ the reaction function for the probability z of the platform regulatory authority choosing strict supervision (an externally decreasing function) is defined as:

$$z=\begin{cases}1 & if\ x<\dfrac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}\\ [0,1] & if\ x=\dfrac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}\\ 0 & if\ x>\dfrac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}\end{cases}$$

It can be seen that if $x=\frac{\alpha yF_w+C_g-F_w-T_g}{\alpha yF_w-F_w-T_g}$,all states remain in neutral equilibrium. If x is greater than the above threshold, z=1 is the evolutionary stable strategy. If x is less than the threshold, $z=0$ is the evolutionary stable strategy.When x exceeds the threshold and $z=1$ becomes the evolutionary stable strategy, it indicates that a high proportion of high-quality products guarantees product safety. To cut supervision costs, the platform tends to adopt loose supervision. In contrast, when manufacturers rarely produce high-quality cosmetics, complaints from beauty reviewers will rise sharply. Accordingly, the platform will strengthen oversight over manufacturers and implement strict supervision.The diagram of the evolutionary stable strategy for the platform regulatory authority choosing strict supervision is presented below.



$z=z^*$ $z>z^*$ $z<z^*$

Fig. 3 Evolutionary Stable Strategy Diagram for Platform Regulatory Authorities

The volume V_{I_p} of Region I_p represents the probability that the platform regulatory authority adopts strict supervision, and the volume V_{II_p} of Region II_p represents the probability of adopting loose supervision.

$$V_{I_p} = \iint \frac{\alpha y F_w + C_g - F_w - T_g}{\alpha y F_w - F_w - T_g} dxdy = \frac{C_g ln(F_w(\alpha - 1) - T_g) - C_g ln(-F_w - T_g) + \alpha F_w}{\alpha F_w}$$

$$V_{II_p} = 1 - V_{I_p}$$

Corollary: The probability that the platform regulatory authority chooses strict supervision decreases with the growth of strict supervision cost C_g, and increases with the rise of administrative penalty T_g and liquidated damages F_w.

Proof: We calculate the partial derivatives of V_{I_p} with respect to C_g, T_g and F_w respectively, and obtain: $\frac{\partial V_{I_p}}{\partial C_g} < 0$, $\frac{\partial V_{I_p}}{\partial T_g} > 0, \frac{\partial V_{I_p}}{\partial F_w} > 0$ Thus, V_{I_p} is a decreasing function of C_g and an increasing function of T_g and F_w. That is, V_{I_p} rises as C_g declines and T_g, F_w increase.

Conclusion: The above corollary reveals that following the cost-benefit principle, regulators will reduce the frequency of strict supervision to lower operational costs. By contrast, higher liquidated damages for violations and heavier administrative penalties for dereliction of supervision will motivate regulatory authorities to strengthen strict supervision.

3.4 Simulation Analysis

Assume that Enterprise A is a cosmetic manufacturing enterprise. To verify the effectiveness of evolutionary stability analysis, this study assigns numerical values to the model parameters based on realistic industry data. Furthermore, MATLAB 2021 is adopted to substitute the above parameters into the system to simulate the tripartite evolutionary path. The initial values are set to evolve 50 times over time, and the simulation results are presented in the figure.

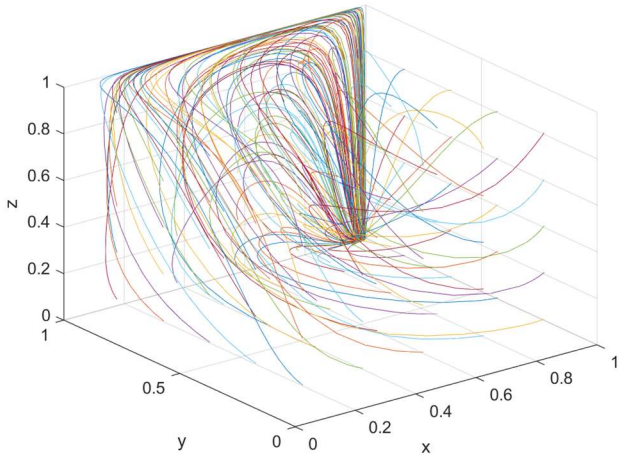


Fig. 4 Tripartite Evolutionary Path Simulation

It can be observed that the system possesses only one evolutionary stable strategy equilibrium point (1, 1, 0), which corresponds to the strategy combination of cosmetic manufacturing enterprises, beauty reviewers, and platform regulatory authorities (producing high-quality cosmetics, providing authentic feedback, and implementing loose supervision). The numerical simulation verifies the correctness of the theoretical analysis. The selection of initial values only affects the strategy convergence rate without influencing the final convergence effect of the system.

4. LITERATURE REFERENCES

In recent years, frequent incidents concerning the quality and safety of cosmetics have drawn widespread attention from all sectors of society. Scholars have conducted extensive research on the supervision and detection of cosmetic quality from diverse perspectives, which can be mainly categorized into three aspects as follows.

First, studies focusing on cosmetic suppliers. Yang, Tsung-Ju et al. [1]adopted questionnaire surveys to explore the decision-making process of beauty reviewers. Their findings revealed that brand image of cosmetics helps improve consumers' perceived product quality and thus enhances customer loyalty. Battie et al. [2]used quality of life scales to empirically verify that the use of cosmetics can facilitate interpersonal relationships, elevate users' quality of life and further boost their sense of well-being. Alani et al.[3] pointed out that although manufacturers make substantial upfront investments in cosmetic R&D and attach great importance to product safety, quality control and testing, adverse reactions caused by product defects or user allergies can hardly be completely avoided. Born et al. [4]indicated that the assessment of the sustainable development of the cosmetics industry is affected by multiple factors, including environmental, social and economic dimensions, as well as the quality and performance of final products. Wirtu [5]stated that harmful chemical substances in synthetic cosmetics may trigger adverse physical reactions, impair human health and even lead to poisoning, allergies and other illnesses. Additionally, such substances pose threats to ecosystems, air quality and marine environments. Goyal et al. [6]emphasized the therapeutic effects of natural plant ingredients in cosmeceuticals. These ingredients can heal skin problems and promote hair growth, with little to no side effects on the human body.

Second, research on the feedback mechanism of beauty reviewers. Lopes et al.[7] applied linear regression to analyze the impact of after-sales services on beauty reviewers' satisfaction. The results showed that post-delivery services and online feedback mechanisms exert positive effects on improving reviewer satisfaction, providing references for enterprises to adjust marketing strategies. Chen[8] quantified the emotional tendency of beauty reviewers' comments on cosmetics across cross-border platforms, and found that a small number of negative comments mainly focused on product packaging, delivery, quality and the lack of free gifts. Investigating reviewers' feedback helps to further understand their preferences and concerns. Chen Y [9]constructed a normative model to study the influence of online reviews by beauty reviewers on sellers' marketing strategies. The research demonstrated that the interaction of product information in reviews is determined by product costs and reviewer types. Sellers can adjust marketing strategies accordingly to increase profits. The study also explored the market conditions for sellers to benefit from reviews and the strategic significance of the timing of review introduction. Vicente[10] found that both monetary and psychological incentives have positive impacts on online word-of-mouth in the cosmetics industry. Meanwhile, demographic and psychological characteristics lead to differentiated effects of incentives among different groups of beauty reviewers.

Third, research on the regulatory system for cosmetic safety. Draelos [11]argued that with the continuous development of chemicals with genuine biological activities, the formulas of cosmetics and skin care products will become increasingly complex, bringing new challenges to government regulators. Given the expanding cosmetics market and growing demand for diversified product ingredients, Zangheri et al.[12] proposed adopting standard analytical methods to quantitatively detect

target substances so as to verify whether cosmetics comply with current regulations. Pauwels[13] noted that cosmetics are subject to both vertical and horizontal legislations. During negotiations with manufacturers and suppliers, the latter are able to determine the type and quantity of data related to the supervision of specific cosmetic ingredients. Lip cosmetics containing mineral raw materials may carry trace heavy metals. Since women use such products frequently, long-term exposure to these chemicals will pose health risks. In view of this, Dahiya et al. [14] held that strict supervision is essential to protect the rights and interests of beauty reviewers and maintain social stability. Bialas et al.[15] suggested that once adverse reactions such as allergies occur after cosmetic use, risk assessment and allergenicity testing of product ingredients should be conducted via methods recognized by scientific and regulatory authorities. Walker[16] concluded that regulatory incentives for cosmetics can effectively promote the allocation and utilization of chemical testing resources. In the research on adverse cosmetic reactions in the European Union, Butschke [17]identified abnormal risk signals through the vigilance notification system to predict cosmetic products that may endanger public health. Rai et al. [18]compared the cosmetic regulatory frameworks of Canada, the United States, the European Union and India based on regulatory databases. The results showed that despite numerous differences in national regulations, all regions share the common goal of delivering high-quality cosmetic products. Against the backdrop of harmonized cosmetic regulations and a well-established regulatory framework in the EU, Zakaria [19]proposed that ASEAN countries represented by Malaysia should introduce additional regulatory requirements to raise public awareness of hazardous ingredients in cosmetics.

5. SUMMARY

For cosmetic manufacturers, enterprises should consciously abide by national safety laws and regulations and refrain from illegal and irregular behaviors. It is essential to implement rigorous full-process quality control, covering raw material procurement, production procedures, and product delivery. Comprehensive quality inspection and supervision should be carried out throughout the whole process to ensure that all products comply with national and industrial quality standards. Furthermore, cosmetic enterprises should attach importance to the cultivation of professional research and development teams to develop innovative products with unique efficacy. Guided by consumer demand, enterprises can develop targeted cosmetics for consumer groups with different skin types, age groups, and personalized needs, such as anti-wrinkle, moisturizing, whitening, and sunscreen products. Meanwhile, enterprises need to improve the customer service system, provide high-quality pre-sale, in-sale, and after-sale services, and ultimately enhance customer satisfaction and brand loyalty.

For beauty reviewers, it is necessary to actively participate in the evaluation of cosmetic product quality and safety and safeguard their legitimate rights and interests through legal means. Cosmetics should be purchased through formal channels such as large shopping malls, brand stores, and officially authorized e-commerce platforms, rather than unqualified offline stores, street stalls, or unknown online platforms, so as to reduce the risk of buying counterfeit and inferior products. In addition, reviewers should select suitable cosmetics according to their actual skin conditions, age, and personal needs, instead of blindly following celebrity endorsements, online influencer recommendations, or trending products. They should stay alert to exaggerated and false publicity of cosmetic products and refuse to believe misleading claims such as “instant effect”, “absolute safety”, and “zero side effects”, so as to rationally recognize that no cosmetic product can achieve immediate and drastic improvement.When beauty reviewers encounter product quality problems or suffer rights infringement, they shall first negotiate with merchants to request product return, exchange, and compensation for losses. During the negotiation, reviewers should remain calm and rational, clearly put forward legitimate demands, and present valid evidence. If the negotiation fails, they can file complaints and reports to market supervision authorities and relevant industry associations with

detailed shopping vouchers, product photos, and problem descriptions to facilitate official investigation and handling.

For platform regulatory authorities, more efficient and economical supervision methods such as risk classification management, targeted key supervision, and random inspections should be adopted to reduce overall supervision costs. Regulators can appropriately increase fines for enterprises producing inferior and low-quality cosmetics, making the penalty cost exceed the illegal profit of producing substandard products, so as to effectively raise enterprises' violation costs. Meanwhile, authorities can set reasonable incentive subsidies for enterprises that produce high-quality cosmetics, making the reward close to their high-quality production costs, thereby fully motivating enterprises to adhere to standardized production.

In addition, it is necessary to strengthen supervision and punishment on irregular behaviors of beauty reviewers. Malicious negative reviews and illegal evaluation behaviors shall be punished in accordance with the law to form effective regulatory deterrence. Moreover, targeted measures should be taken to increase the rent-seeking costs of cosmetic manufacturers, such as requiring enterprises to regularly disclose operational information including raw material sources, procurement records, production processes, and quality control conditions, so as to fundamentally standardize the market operation of the cosmetics industry.

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